Dynamic Energy Management: a Practical Approach to Grids and Renewable Integration

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This talk

- network of production, distribution, and use of energy
  - basic abstractions: *devices*, *terminals*, *nets*
  - operate network by minimizing costs, satisfying conservation of energy
- *static*, *dynamic*, *uncertain*, and *robust* optimization
- realistic and practical models
- examples with real data: micro-grid, wind power plant, ...

References

- Dynamic Energy Management, *N. Moehle, E. Busseti, S. Boyd, M. Wytock*, Large Scale Optimization in Supply Chain & Smart Manufacturing, 2019
- `cvxpower`, open-source Python implementation, `https://github.com/cvxgrp/cvxpower`

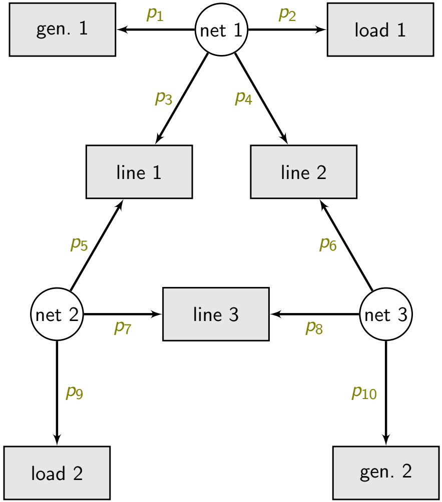
- we use standard techniques
  - *Convex Optimization*, S. Boyd and L. Vandenberghe, Cambridge University Press, 2004
  - *Model predictive control design: New trends and tools.*, A. Bemporad, IEEE Conference on Decision and Control, 2006

Outline

Basic abstractions

- **devices** produce, distribute, or use energy
  - power plant, grid link, ...
  - transmission line, transformer, rectifier, inverter, ...
  - home, factory, office building, ...
- each has one or more **terminal** to exchange energy
  - of certain type: DC, AC, different voltage, ...
- terminals are connected to **nets**
  - conservation of energy
- devices connected via terminals to nets form a **network**

Three-bus example network



Power flow and cost functions

- power flow vector $p \in \mathbf{R}^M$, M is number of terminals
- negative if energy leaves device, positive if enters
- p_d is subvector of p for the terminals of device d
- $f_d(p_d) \in \mathbf{R}$ is cost function (*e.g.*, in €) of device d
- $f(p) = \sum_d f_d(p_d)$ is cost function of the whole network

Conservation of energy

- energy is conserved at each of the N nets
- topology described by *adjacency matrix* $A \in \mathbf{R}^{N \times M}$

$$A_{nm} = \begin{cases} 1 & \text{terminal } m \text{ connected to net } n \\ 0 & \text{otherwise} \end{cases}$$

- vector equality constraint

$$Ap = 0$$

Optimal power flow

- *static optimal power flow* optimization problem

$$\begin{array}{ll} \text{minimize} & f(p) \\ \text{subject to} & Ap = 0 \end{array}$$

- solution $p^\star$ gives lowest cost feasible network operation
- convex optimization problem (if f is convex)
  - solved efficiently and reliably in little time
  - distributed solution possible
- *Lagrangian* multiplier $\lambda \in \mathbf{R}^N$ as by-product of solution

Prices and payments

- λ equal to the *locational marginal prices* at the nets
  - marginal cost of energy at each net
- payment to device d

$$-\lambda_d^T p_d^\star$$

  - can be used for internal accounting or between operators
- device P&L

$$-\lambda_d^T p_d^\star - f_d(p_d^\star)$$

- optimality conditions imply that $p^\star$ maximizes each device's profit
- global and individual *optima* coincide!

Device examples I

- generator

$$f_d(p_d) = \begin{cases} \phi_d(-p_d) & p_{\min} \le -p_d \le p_{\max} \\ \infty & \text{otherwise} \end{cases}$$

  - ϕ quadratic for conventional generator
  - $\phi(p) = 0$ for renewable with $p_{\max} = p_{\text{avail}}$

- fixed load

$$f_d(p_d) = \begin{cases} 0 & p_d = p_{\text{fixed}} \\ \infty & \text{otherwise} \end{cases}$$

  - also flexible load

Device examples II

- grid tie

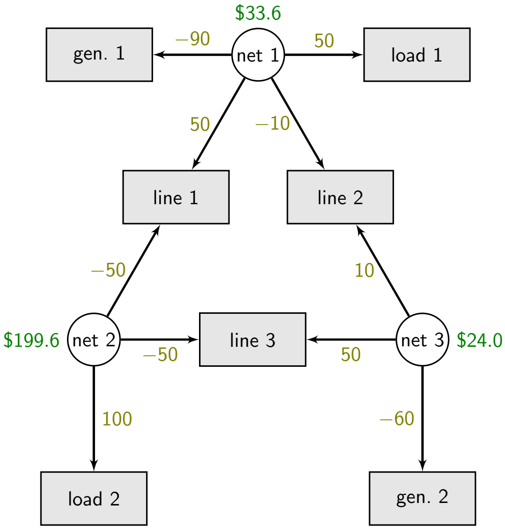
$$f_d(p_d) = \max\{-\lambda_{\mathrm{buy}} p_d, -\lambda_{\mathrm{sell}} p_d\}$$

- lossless transmission line, $p_d = (p_1, p_2)$

$$f_d(p_1, p_2) = \begin{cases} 0 & p_1 + p_2 = 0 \text{ and } p_{\min} < p_1 < p_{\max} \\ \infty & \text{otherwise} \end{cases}$$

  - can also model quadratic power loss

Three-bus example network with solution



Outline

Dynamic optimal power flow

- $T > 0$ time periods (*e.g.*, 15 minutes intervals)
- *power flow schedule* $p \in \mathbf{R}^{M \times T}$
- device cost function $f_d : \mathbf{R}^{M_d \times T} \to \mathbf{R}$
- $f(p) = \sum_d f_d(p_d)$
- *dynamic optimal power flow* optimization problem

$$\begin{array}{ll} \text{minimize} & f(p) \\ \text{subject to} & Ap = 0 \end{array}$$

- identical to static problem, with matrix notation
- multiplier $\lambda \in \mathbf{R}^{N \times T}$ gives prices at nets in time
  - used to define *payment rates* of devices

Device examples I

- static devices' cost functions summed over time
  - fixed load

$$f_d(p_d) = \begin{cases} 0 & p_{d,t} = p_{\mathrm{fix},t}, \quad t = 1, \ldots, T \\ \infty & \text{otherwise} \end{cases}$$

- devices with *time coupling*
  - add smoothness penalty $\sum_{t=0}^{T-1} \phi\left(p_{d,t+1} - p_{d,t}\right)$
    - ramp constaint

$$\phi_{\text{ramp}}(x) = \begin{cases} 0 & -r_{\text{down}} \leq x \leq r_{\text{up}} \\ \infty & \text{otherwise,} \end{cases}$$

Device examples II

- deferrable load, total energy E_{def}

$$f_d(p_d) = \begin{cases} 0 & \sum_{t=s}^{e} h p_{d,t} = E_{\mathrm{def}}, \quad 0 \le p_{d,t} \le p_{\max} \\ \infty & \text{otherwise} \end{cases}$$

- thermal load
  - room (or refrigerator, or building) temperature θ_t
  - $f_d(p_d) = 0$ if, for all t:
    - $\theta_{\mathrm{min}} \le \theta_t \le \theta_{\mathrm{max}}$
    - $\theta_{t+1} = \theta_t + (\mu/c)(\theta_{\mathrm{amb}} - \theta_t) - (\eta/c)p_{d,t}$
    - $0 \le p_{d,t} \le p_{\mathrm{max}}$
  - $f_d(p_d) = \infty$ otherwise

Device examples III

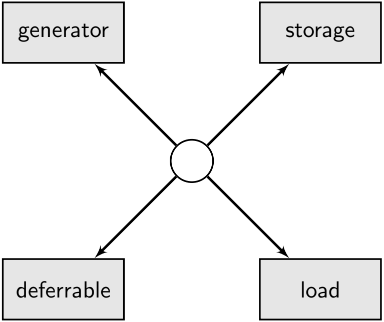
- storage device
  - $f_d(p_d) = 0$ if, for all t:
    - $E_{t+1} = (1-\alpha)E_t + hp_{d,t}$
    - $E_{\min} \leq E_t \leq E_{\max}$
    - $p_{\min} \leq p_d \leq p_{\max}$
  - $f_d(p_d) = \infty$ otherwise
- can add *charge/discharge* cost to f_d

$$\frac{Ch}{2n_{\mathrm{cyc}}(E_{\max} - E_{\min})} \sum_{t=1}^{T} |p_{d,t}|$$

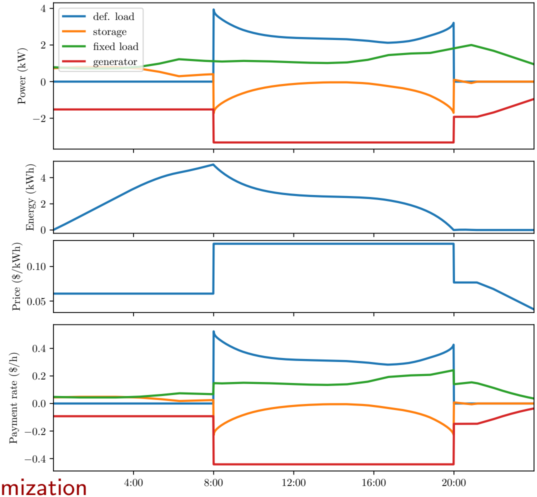
  - amortization of capital investment C over n_{cyc} cycles

Home energy example I

- off-grid home, single net
- optimize over 24 hours

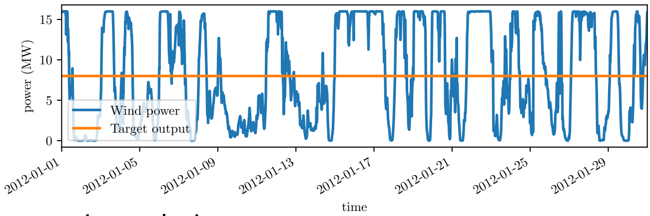


Home energy example II



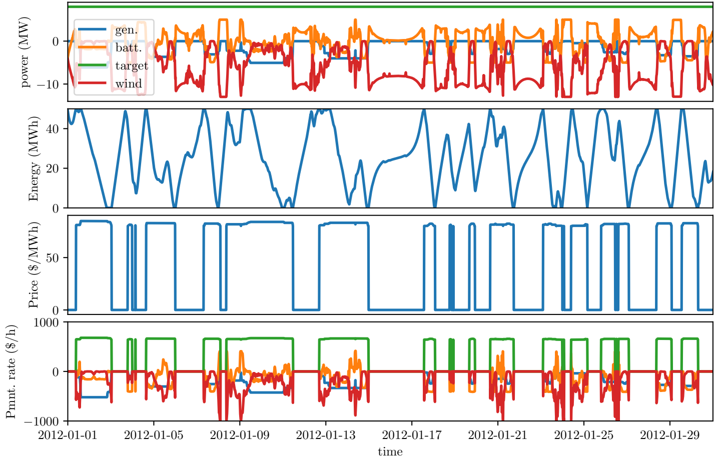
Example: wind power plant I

- model of wind power plant
  - wind turbine, storage unit, small conventional generator, fixed load
- operate over a month
- highly volatile wind power (real data)



- *prescient* solution
  - lowest achievable cost

Example: wind power plant II



Outline

Model predictive control

- optimize controls of dynamic system in time
- use predictions of future values
  - loads
  - renewable available power
  - grid power prices
- implement first decision
  - future decisions are *planning exercise*
- re-optimize at next period, with updated predictions
- industrial engineering standard for digital control

Uncertain optimal power flow

- plan out $T > 0$ steps in the future
  - *receding horizon* control
- like dynamic optimal power flow, repeated at $t = 1, 2, \ldots$
  - planned power flow schedule $p_{|t} \in \mathbf{R}^{m \times T}$
  - predicted network cost function $\hat{f}_{|t} : \mathbf{R}^{M \times T} \to \mathbf{R}$
- *uncertain optimal power flow* problem

$$\begin{array}{ll} \text{minimize} & \hat{f}_{|t}(p_{|t}) \\ \text{subject to} & Ap_{|t} = 0 \end{array}$$

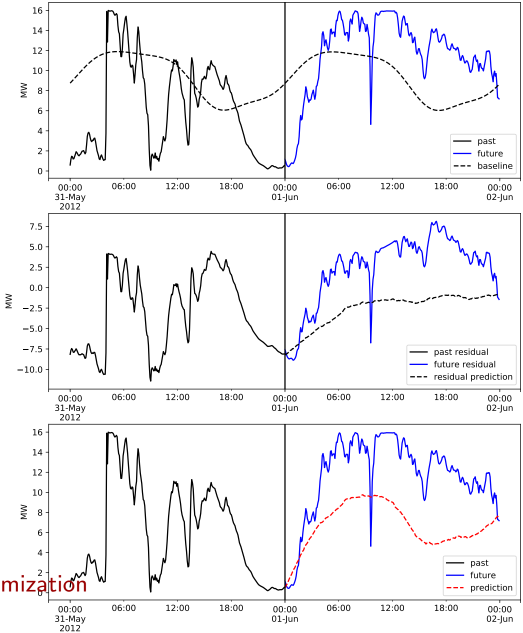
- multiplier $\lambda \in \mathbf{R}^{N \times T}$ is price schedule
  - first column are current prices, others are predictions
  - used to define payment scheme

Example: wind power plant II

- predictive model for available wind power
- predict next 24 hours
  - seasonal (daily and annual) effects
  - autoregression on last 24 hours

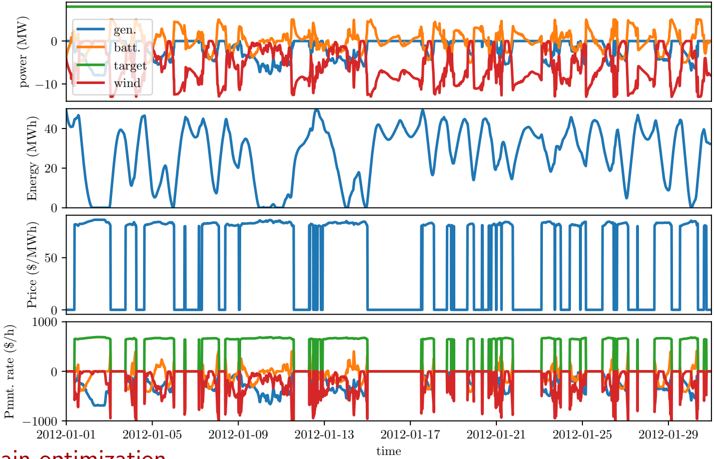
- simple prediction is accurate in the near future
  - that's all we need

Example: wind power plant IV



Example: wind power plant V

- visibly similar to prescient solution



Outline

Robust optimal power flow I

- variant of MPC
- model $S > 0$ *scenarios* for future unknowns
  - scenario s is realized with probability π_s
  - $\sum_s \pi_s = 1$
- plan a power flow for each
  - scenario power flow schedules $p_{|t} \in \mathbf{R}^{M \times T \times S}$
- cost function $f_t : \mathbf{R}^{M \times T \times S} \to \mathbf{R}$
  - expected value of scenario cost functions

$$f_t(p_{|t}) = \sum_s \pi_s f^{(s)}(p_{|t}^{(s)})$$

Robust optimal power flow II

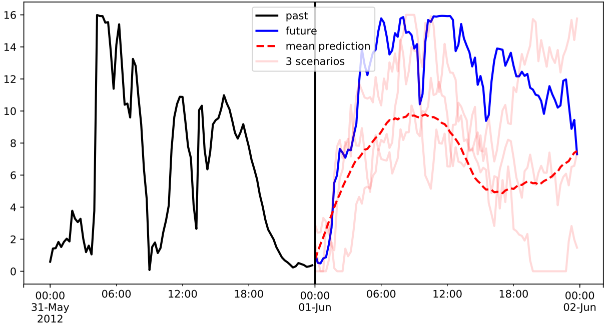
- require that first elements of power flow coincide
  - $p_{|t}^{\mathrm{nom}} \in \mathbf{R}^M$
- *robust optimal power flow* problem

$$\begin{array}{ll} \text{minimize} & \hat{f}(p_{|t}) \\ \text{subject to} & Ap_{|t}^{(s)} = 0 \quad s = 1, \ldots, S \\ & p_{t+1|t}^{(s)} = p_{|t}^{\mathrm{nom}} \quad s = 1, \ldots, S, \end{array}$$

- multiplier $\lambda \in \mathbf{R}^{N \times T \times S}$ is predicted prices under each scenario

Example: wind power plant VI

- same predictive model
- generate future scenarios using typical forecast errors
  - each is a *plausible future*
  - use these to define scenario cost functions



Example: wind power plant VII

- cost equivalent to prescient solution!!